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LETTER TO THE EDITOR

Treatment of the 3D Ising gauge model by a Monte Carlo renormalisation group method

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Abstract. The Ising gauge model on a simple cubic lattice with finite extension in one direction is treated by Binder's Monte Carlo renormalisation group method. β/ν is found to increase with increasing number of layers to values larger than $\frac{1}{8}$.

In the early seventies the Ising gauge model was introduced by Wegner (1971) in his work on duality relations between general Ising models. The three-dimensional Ising gauge model on a simple cubic lattice is dual to the three-dimensional ordinary Ising model (Balian *et al* 1975). Its Hamiltonian is

$$-H/k_{\rm B}T = a\sum_{\rm plaq}\sigma_{\rm p}. \tag{1}$$

The variables are placed on the links between the lattice points, $\sigma_{\rm p}$ means the product around an elementary plaquette, i.e. around the smallest connected loop of four links ($\sigma_{\rm p} = \sigma\sigma\sigma\sigma$, $\sigma = \pm 1$).

Since the Ising gauge model is one of the simplest models of lattice gauge theory (Wilson 1974), most progress in the investigation of its statistical properties has been made within the framework of theoretical high energy physics. In this letter I want to present some new results from the point of view of statistical physics. The conclusions for high energy physics will be published elsewhere.

Due to the local gauge symmetry of the Ising gauge model (flipping all spins placed on links joined to one lattice point does not change the interaction energy) it avoids, at a first glance, the usual methods of statistical physics developed for models with global symmetries. In particular, since the local gauge symmetry cannot be spontaneously broken (Elitzur 1975) there does not exist a local order parameter corresponding to this symmetry. Instead, the two phases are usually characterised by the differences in the long-range behaviour of a gauge invariant correlation function, the so-called Wilson loop (see Kogut 1979).

However, if the lattice is finite in one direction (say the vertical direction) and the boundary conditions in this direction are periodic, it is possible to define an order parameter which indicates the spontaneous breaking of a global Z(2) symmetry analogous to the magnetisation in models of solid state physics (Kuti *et al* 1981, McLerran and Svetitsky 1981, Weinkauf and Zittartz 1982). The transformation according to this symmetry, the so-called centre symmetry (Susskind 1978, Polyakov 1978), is performed by flipping the spins placed on all vertical links between a single pair of

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adjacent horizontal layers. The order parameter, the Polyakov loop correlation function, is defined as

$$\langle L\rangle = \lim_{h\to 0}\lim_{N_S\to\infty}\left\langle \prod_{j\in\Gamma}\sigma_j\right\rangle \tag{2}$$

where the σ_j belong to vertical links which form a loop Γ closed due to the periodic boundary conditions. N_S and N_T define an $N_S\times N_S\times N_T$ lattice and h denotes an external field coupled to the Polyakov loops through an energy

$$-hL = -h\prod_{j\in\Gamma}\sigma_j. \tag{3}$$

Recently many papers have been published concerning the behaviour of $\langle L\rangle$ close to the transition point in the Ising gauge model (and other lattice gauge models where L is defined similarly). The latest of these papers deal with a conjecture made by Svetitsky and Yaffe (1982) for a whole class of lattice gauge models. It says that the effective interaction between Polyakov loops, which is obtained when all degrees of freedom except the Polyakov loops are integrated out, is short ranged. For the Ising gauge model, Svetitsky and Yaffe concluded that due to the global Z(2) symmetry the resulting two-dimensional effective theory for Polyakov loops should belong to the universality class of the two-dimensional Ising model. In particular, the critical exponents should be

$$\beta = \tfrac{1}{8}, \qquad \nu = 1.$$

The validity of this conjecture has been examined by mean-field calculations (Gross and Wheater 1984a, Matsuoka 1984, Alessandrini and Boucaud 1984, Green and Karsch 1984), strong coupling expansions (Polonyi and Szlachanyi 1982, Gross 1983), and various Monte Carlo calculations (Celik *et al* 1983, Gavai and Karsch 1983, Gavai and Satz 1985, Gross and Wheater 1984b, Curci and Tripiccione 1984, Wansleben 1984). Most of these publications support the conjecture whereas some Monte Carlo results are not consistent with it (Wansleben 1984, Gross and Wheater 1984b). In the (2+1)-dimensional Ising gauge model with $N_T=4$, β was found to be $0{\cdot}20\pm0{\cdot}04$ (Wansleben 1984). $\nu=1$ is known from the duality to the three-dimensional Ising model which holds exactly in the case of finite extension in one direction with periodic boundary conditions.

The common difficulty of all Monte Carlo calculations in this field which have come to my knowledge up to now is that a fit of the data to

$$\langle L\rangle = B_1(T-T_c)^{\beta}[1+B_2(T-T_c)^{\Delta}+\ldots] \tag{4}$$

(Wegner 1972) requires the determination of all five parameters (the critical temperature is unknown). If the correction term is ignored, systematic errors in the resulting leading exponent β are expected. Therefore, a reliable error estimate is difficult, even when a finite-size scaling analysis is performed which gives a check whether a chosen set of parameters is consistent with finite-size scaling theory (Wansleben 1984).

One way to avoid this basic difficulty is the use of Monte Carlo renormalisation group (MCRG) methods which yield the critical temperature independently of the determination of critical exponents. The basic idea of most of these methods, the blocking of spins into block spins, can be applied only in those directions where the lattice is infinite. This means that a MCRG procedure for the effective theory of Polyakov loops must be carried out. Since the Hamiltonian of this theory is unknown so far (and will contain many different types of interactions) the widespread method of Swendsen (Swendsen 1982, Pawley *et al* 1984) is not suitable for this problem. An alternative method is that of Binder (1981). It is based on the treatment of the block spins defined as

$$s = (1/l^2) \sum_{j \in \text{cell}} L_j$$

i.e. the average over a cell of length l. The critical temperature is determined by the non-trivial fixed point of the fourth cumulant of the block-spin distribution function,

$$U_l = 1 - \langle s^4 \rangle_l / 3 \langle s^2 \rangle_l^2,$$

regarded as a function of l (figure 1). $\langle\ \rangle_l$ means the expectation value for block spins of size l. From a finite-size scaling ansatz for the block-spin distribution function one finds (Binder 1981)

$$2\beta/\nu = W_\beta \tag{5}$$

with

$$W_\beta = -(1/\ln b) \ln(\langle s^2 \rangle_{bl} / \langle s^2 \rangle_l) \tag{6}$$

at the non-trivial fixed point of U_l.

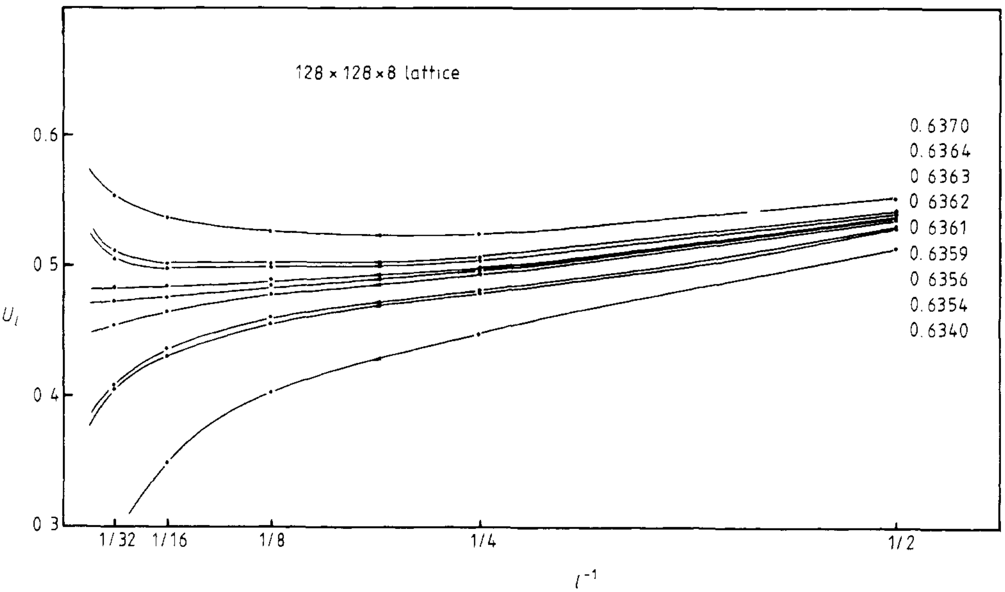

Figure 1. Flow of U_l as a function of l for $N_T = 8$. tanh(a), with a = (interaction energy)/k_BT, corresponding to the different trajectories is given by the numbers on the right-hand side within the frame. The trivial fixed points are $U_l = 0$ and $U_l = \frac{2}{3}$.

Unfortunately, equation (5) holds only if corrections to scaling are neglected; however, they seem to be very important here. Binder found that the correction to scaling term which must be supplemented at the left-hand side of equation (5) asymptotically ($b \to$ infinity) goes like $(\ln b)^{-1}$ for l fixed (figure 2). A systematic extrapolation to infinite l at fixed b (Kalle 1984) was not carried out by Binder and

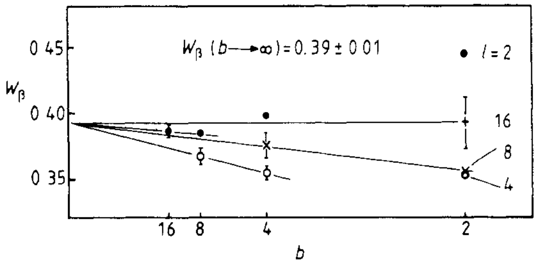

Figure 2. Extrapolation of W_β to infinite b for $N_T = 8$ at $\tanh(a) = 0.6360$. The scale of the abscissa is the inverse logarithm of b.

seem to be impossible with my data. Thus my results may contain some systematic error which will be discussed later. Analogous considerations lead to a $(\ln b)^{-1}$ dependence of the critical temperatures determined by the condition $U_l = U_{bl}$ (figure 3).

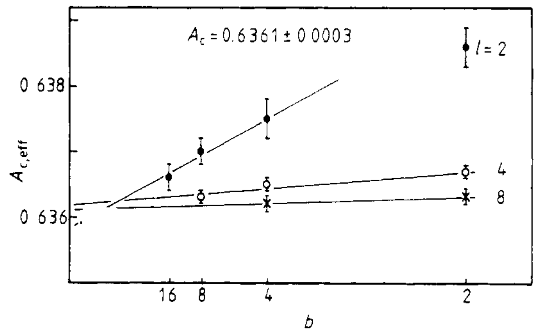

Figure 3. Extrapolation of the effective critical constants determined by the matching condition $U_{bl} = U_l$ for $N_T = 8$. $A = \tanh(a)$. The scale of the abscissa is the inverse logarithm of b.

I simulated an $N_S \times N_S \times N_T$ lattice with $N_S = 128$ and $N_T = 2, 4, 8$. I used a fully vectorised multispin coding algorithm (Wansleben *et al* 1984) on a two-pipe CDC Cyber 205 including a shift-register random number generator (Kalle and Wansleben 1984). The speed is about 15 million updates per second for $N_T = 8$.

The number of iterations at given temperature was 460 000 for $N_T = 2, 4$ and 350 000 for $N_T = 8$. In addition for every N_T at least two runs with 1.5 million Monte Carlo steps per spins were carried out with temperatures close to the critical ones. W_β was determined for several temperatures around the critical point and extrapolated to infinite b as shown in figure 2. These values are plotted against the temperature in figure 4 for $N_T = 8$. The effective critical temperatures determined by the matching condition $U_{bl} = U_l$ were extrapolated to infinite b as shown in figure 3. The final result for β/ν can be read from figure 4 (for $N_T = 8$). It is the value of $W(\infty)/2$ at the critical temperature ($\beta/\nu = 0.19 \pm 0.03$ for $N_T = 8$).

The results are summarised in table 1. The given error bars are standard deviations arising from statistical errors. These values may have some systematic error in addition. First, the vaues for W_β extrapolated to infinite b at a given temperature (figure 2) might be underestimated because a systematic dependence on l may not have fully

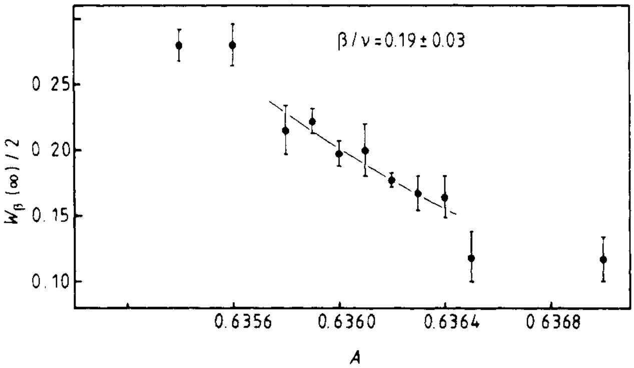

Figure 4. W_β extrapolated to infinite b plotted against $A = \tanh(a)$ for $N_T = 8$. The critical temperature is given by $A = 0.6361 \pm 0.0003$.

Table 1. Final results for various numbers of layers N_T. $\nu = 1$ is known from duality arguments.

N_T	β/ν
2	0.15 ± 0.03
4	0.16 ± 0.03
8	0.19 ± 0.03

cancelled out when I used this method of extrapolation (note that the values of W_β at fixed b for $l > 2$ increase with increasing l). Second, the critical temperatures may be overestimated due to the finite length of the whole lattice. Taking into account these two points and the behaviour of W_β as a function of the temperature (figure 4), I conclude that my estimates for β/ν may be too low due to higher-order corrections to scaling. Thus, the MCRG treatment as presented here with β near 0.17 is consistent with the results of standard Monte Carlo simulation where $\beta = 0.20 \pm 0.04$ for $N_T = 4$ was found (Wansleben 1984). Moreover, it suggests a slight increase of β/ν with increasing N_T. The question remains open, however, whether a limit of N_T to infinity is a reasonable limit for β.

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